**KEPLER’S LAWS AND THE ORBIT OF MERCURY**

**Kepler’s Laws state:**

**1. The planets move in elliptical orbits with the Sun at one focus**

**2. A radius vector from the Sun to a planet sweeps out equal areas in equal periods of time**

**3. The ratio of the cube of the average distance of any planet from the Sun to the square of its period of revolution is a constant (P2 = a3)**

**Materials:**

graph paper, ruler, protractor

**Purpose:**

1. Plot the distance of Mercury from the Sun in Astronomical Units (AU) for equal time intervals, to derive the shape of Mercury’s orbit;

2. Measure the area swept out by radius vectors at perihelion (closest approach) and aphelion (farthest approach) and demonstrate that the areas swept out in equal time periods are equal;

3. Use the distance from the Sun and orbital period data for the solar system as a whole to derive the relationship between the period and semi-major axis of any planet.

**Activity 1: Given the distance from the Sun of 18 positions of the planet Mercury, plot the orbit of Mercury around the Sun.**

Mercury makes one complete orbit around the Sun in 88 days. There are 18 observations; five days separate each observation, except the last one between 18 and 1, which is only a three-day interval. The distance measurements are measured from the center of the Sun to the center of Mercury, and the units are “AU” for Astronomical Units. One AU is the average distance between the Sun and the Earth, which is equal to approximately 1.5 x 108 km (150,000,000 km).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Observation #** | **Degrees** | **Distance from Sun (A.U.)** | **Observation #** | **Degrees** | **Distance from Sun (A.U.)** |
| **1** | 4 | .35 | **10** | 224 | .45 |
| **2** | 31 | .32 | **11** | 239 | .46 |
| **3** | 61 | .31 | **12** | 252 | .47 |
| **4** | 92 | .31 | **13** | 266 | .47 |
| **5** | 122 | .32 | **14** | 280 | .46 |
| **6** | 149 | .35 | **15** | 295 | .44 |
| **7** | 172 | .38 | **16** | 311 | .42 |
| **8** | 192 | .41 | **17** | 330 | .40 |
| **9** | 209 | .43 | **18** | 350 | .37 |

**Procedure:**

1. Holding a piece of graph paper sideways, put a dot for the Sun a distance about 5 inches from the bottom (This choice of where to put one focus is so that your ellipse will fit on the paper.)

2. Using a protractor, plot the positions of Mercury relative to the Sun, according to the data table. Start with 00 on the right, and go counter clockwise. Use a scale of 1 inch = 0.1 AU, or 2.5 cm = 0.1 AU.

3. Connect the dots as smoothly as possible. You now have the orbit of Mercury.

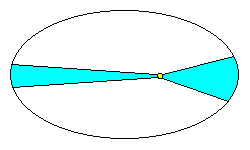
Notice the distance from the Sun to Mercury is not constant, making the orbit slightly elliptical. In fact, if you erase for a moment the dot you made for the Sun, and only look at the path of Mercury, the orbit looks almost circular.

Pluto has the most eccentric (elliptical) orbit, sometimes even coming inside the orbit of the farthest out planet, Neptune.

**Activity 2: Verify Kepler’s Second Law using the graph and data from Part 1. See if the area of a sector swept out by a vector from the Sun to Mercury remains constant between any two data points separated by equal periods of time.**

**Procedure**:

1. Select two points that are closest to the Sun (I suggest 3 and 4) and two that are farthest from the sun (I suggest 12 and 13), as suggested in the figure below.



2. Draw the radius vector from the Sun to each data point. Call the short, thick sector “A” and the long, thin sector “B” (or whatever label you want to give them).

3. Label the distance in AU to each data point. Find the average radius for each sector as

raverage = (rx + ry)/2, and measure the central angle, Θ, for each sector.

4. Compute the approximate area of each sector as: (Θ/360) x π (raverage)2

**Sector A Sector B**

raverage = (rx + ry)/2 = ­­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_ raverage = (rx + ry)/2 = ­­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Θx-y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Θx-y  = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

area = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ AU2 area = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ AU2

**Questions**:

1. Compare the areas of sectors A and B. Are they approximately equal, at least within your experimental error?

2. When is Mercury moving faster, when it is closest or farthest from the Sun? Give an explanation for your answer.

**Activity 3. Period – distance data, or Kepler’s Law of Harmonies**

Using data for the solar system as a whole, you will demonstrate Kepler’s Third Law, which he called his Law of Harmonies.

Here are the data for semi-major axis of the orbit of each planet relative to the Sun in AU, and their orbital periods in years:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Planet** | **semi-major axis (AU)**  **a** | **Period (years)**  **p** | **a3** | **p2** |
| **Mercury** | .386 | .241 |  |  |
| **Venus** | .720 | .615 |  |  |
| **Earth** | 1.00 | 1.00 |  |  |
| **Mars** | 1.52 | 1.88 |  |  |
| **Jupiter** | 5.19 | 11.9 |  |  |
| **Saturn** | 9.53 | 29.5 |  |  |
| **Uranus** | 19.1 | 84 |  |  |
| **Neptune** | 30.0 | 165 |  |  |
| **Pluto** | 39.3 | 248 |  |  |

Procedure:

1. Graph the length of the semi-major axis (a) in AU on the y-axis, and the orbital period (P) in years on the x-axis.

What is the shape of this graph?

2. Calculate the cube of the semi-major axis (a3) and the square of the period (p2); write your calculations in the table.

3. Graph a3 on the y-axis and p2 on the x-axis.

What is the shape of this graph?